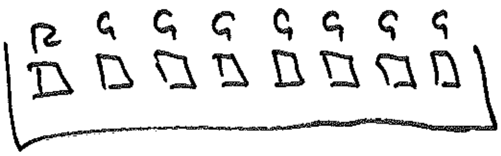
probability. — how to avoid counting

toss a coin 6 times, what's the chance of exactly 2 heads?.



Five draws with replacement what's the chance of exactly two of the draws being R?

R R G G G ← one way of obtaining exactly 2 R.

$$\frac{1}{8} \times \frac{1}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$$

R G R G G ← 9 other ways of obtaining exactly 2 R.

R G G R G

R G G G R

G R R G G

G R G R G

G R G G R

G G R R G

G G R G R.

G G G R R.

10 ways of getting 2R + 3G.

The 10 options are mutually exclusive.

<u>Recall</u> : addition rule.

Probability of any one out of the 10 is the <u>sum</u> of the individual probabilities. (because they are mutually exclusive).

$$P(2R + 3G) = 10 \times \left(\frac{1}{8}\right)^2 \left(\frac{7}{8}\right)^3$$

10 different ways of getting 2R+3G

2reds

3 greens

Q: can we avoid listing all the possibilities + counting them?

A: the number of possibilities is given by the <u>Binomial Coefficient</u>.

Binomial Coefficient.

Counts the number of ways you can arrange indistinguishable objects of two different colours.

If we have 5 distinct objects, there are

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

different ways of arranging them.

If 3 of the objects are G, and are indistinguishable from each other, these 3 objects can be arranged in

$$3 \times 2 \times 1 = 6$$

ways.

If 2 of the objects are R, and indistinguishable from each other, they can be arranged in

$$2 \times 1 = 2$$

ways

$\Rightarrow$ Total # of ways of arranging 3 G and 2 R is

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1) \times (2 \times 1)} = 10$$

If we have 1R + 4G

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1) \times (1)} = 5$$

$n!$ "n factorial"

$$n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$1! = 1 = 1.$

$0! = 1$ (by definition).

$\Rightarrow$ # ways of arranging 3G + 2R is

$$\frac{5!}{3!\,2!}$$

ways of arranging 4G + 1R is

$$\frac{5!}{4!\;1!}.$$

ways of arranging 5G + 0R is

$$\frac{5!}{5!\;0!} = 1.$$

Binomial Coefficient.

n draws.

k "sucesses"

$$\frac{n!}{k!\,(n-k)!} \quad \text{written as} \quad \binom{n}{k}$$

"n chose k"

this is the number of different ways we can arrange n objects, where k are of one type, and $(n-k)$ are of the other type.

Binomial Probability

If each success has probability p
failure $1-p$

Then the probability of k sucesses in n trials is

$$\frac{n!}{k!\,(n-k)!}\, p^k\,(1-p)^{n-k}$$

provided that:

- n is fixed in advance.
- p is the same for each trial.
- trials are independent.

Example.

A family has 4 children.
What's the chance they have more girls than boys?

Assume that each child has $p = 1/2$ of being a girl, and that each child is independent of the others.

Chance of more girls than boys

$p($ 3 out 4 girls$)$ + $p($ 4 out of 4 girls$)$.

(mutually exclusive events).

$$\frac{4!}{3!\ 1!} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^1$$

$$+ \frac{4!}{4!\ 0!} \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^0$$

$= 31\%$ (0.31).

11 students are from families with 4 kids.

3 ~~have~~ are from families with 3 or 4 girls.

$\frac{3}{11} = 0.27$ ← <u>close</u> [illegible]

Uk National Lottery.

49 numbered balls.

chose 6.

chance of winning

$$\frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44} = \frac{1}{13,983,816}$$

1 in 14 million.

(ie ~~you ca~~ if you buy 1 ticket/week,
you expect to win once every
~ 269,000 years)

* what's the chance of one particular ball (eg 23)
coming up in a given weekly draw?

$$\frac{6}{49}.$$

In a large number of games, how often should we expect this ball to appear?

How much variability should we expect?