Probability.

$$P(\text{win}) = P\left(\begin{array}{l}\text{first ball drawn} \\ \text{is one of your chosen numbers}\end{array}\right).$$

$$\times\, P\left(\begin{array}{l}\text{2nd ball drawn is one of} \\ \text{your remaining chosen numbers}\end{array} \,\middle|\, \begin{array}{l}\text{1st ball drawn being} \\ \text{one of your chosen} \\ \text{numbers}\end{array}\right).$$

$$\times\, P\left(\begin{array}{l}\text{3rd ball drawn is one of} \\ \text{your remaining chosen numbers}\end{array} \,\middle|\, \begin{array}{l}\text{1st + 2nd balls were} \\ \text{two of your chosen numbers}\end{array}\right)$$

$$\vdots$$

$$\times\, P\left(\begin{array}{l}\text{5th ball drawn is your} \\ \text{last chosen number}\end{array} \,\middle|\, \begin{array}{l}\text{1st, 2nd, 3rd, 4th balls} \\ \text{were 4 of your} \\ \text{chosen numbers}\end{array}\right).$$

$$\times\, P\left(\begin{array}{l}\text{your powerball choice is} \\ \text{the number drawn}\end{array}\right).$$

$$= \frac{5}{69} \times \frac{4}{68} \times \frac{3}{67} \times \frac{2}{66} \times \frac{1}{65} \times \frac{1}{26}$$

$$= \frac{1}{292{,}201{,}338}.$$

$$\approx 0.$$

Note: the ~~draw~~ 1st 5 balls are dependent events
(each drawing depends on all the earlier draws)

The powerball is independent

Examples:

Draw 2 cards off the top of a well-shuffled deck.

prob. that the 2 cards are both aces.

4 out of 52.
aces.

$$P(1^{st} \text{ card is A}) = \frac{4}{52}.$$

given that 1st card is A. 3 A left out of ~~52~~ 51 cards

$$P(2^{nd} \text{ card is A} \mid 1^{st} \text{ card was A})$$

$$= \frac{3}{51}$$

$$P(1^{st} \text{ 2 cards are both A}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652}.$$

Toss coin twice.

what's the chance of HT ?

$P(H \text{ on } 1^{st} \text{ toss}) = 1/2.$

$P(T \text{ on } 2^{nd} \text{ toss}) = 1/2. \quad = P(T \text{ on } 2^{nd} \text{ toss} \mid H \text{ on } 1^{st} \text{ toss}).$

for the coins, what happened on earlier events <u>does not</u> affect later events

- they are independent.

Two events are <u>independent</u> if the chances for the 2nd event given the first are the same, regardless of the outcome of the 1st event.

- otherwise they are dependent.

$\boxed{1}\ \boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}$

two draws, at random, with replacement

1st draw was a 1, what's the chance of $\boxed{2}$ on 2nd draw?

when drawing <u>with</u> rep<u>lacement</u> $\rightarrow$ independent events, the content of the box is the <u>same</u> for every draw.

$$\Rightarrow P(\boxed{2} \text{ on 2nd draw}) = \frac{2}{5}$$

if draws are made <u>without</u> rep<u>lacement</u> $\rightarrow$ dependent events

After $\boxed{1}$ is drawn on 1st draw, box now contains

$\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}$

$$P(\boxed{2} \text{ on 2nd draw} \mid \boxed{1} \text{ on 1st draw}) = \frac{2}{4} = \frac{1}{2}.$$

Some Notation.

event A

probability of event A is written $P(A)$

events A and B

conditional probability of event B given event A $= P(B|A)$

Multiplication Rule.

$$P(A \text{ and } B) = P(B|A)\,P(A)$$
$$= P(A|B)\,P(B)$$

Independence.

requires that

$$P(B|A) = P(B)$$
$$P(A|B) = P(A)$$

implying that $P(A \text{ and } B) = P(A) \times P(B)$

for independent events

Probability of an event $=$ $\dfrac{\#\text{ outcomes corresponding to that event}}{\text{total }\#\text{ outcomes}}$

when each outcome is equally likely.

ie list the possible outcomes.

count the relevant ones.

eg Roll two dice.

6 possible outcomes on 1^{st} die

for each of these, there are 6 possible outcomes on 2^{nd} die.

$\Rightarrow$ 36 possible outcomes in total.

White die

Red die	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

P(sum of the two rolls will be 2)

- only one possibility (1,1)

$\Rightarrow$ prob. is $1/36$

P (sum will be 7)

- 6 possibilities (6,1) (5,2) (4,3) (3,4), (2,5) (1,6).

$\Rightarrow$ prob is $\frac{6}{36} = \frac{1}{6}$

P(1st die has larger # pips than 2nd die)

- this corresponds to all the entries in the bottom-left part of the table
- there are 15 ways that the 1st die shows more spots than 2nd die.

prob is $\frac{15}{36}$

P(Red die showing more pips than white die | white die showing 4 pips).

Restrict our set of possible outcomes to those that fit the condition.

There are now <u>6</u> outcomes that we're interested in, and <u>two</u> of them (5,4) (6,4) have more spots on red than on white

$\Rightarrow$ prob $= \frac{2}{6} = \frac{1}{3}$

Recall. multiplication rule for cases like.

$P($1st ball is blue and 2nd ball is green$)$

Now. look at "or" case.

event A happens or B happens or both

Start by looking at the case where it is impossible for both events to happen together.

Such events are called mutually exclusive

Two events are mutually exclusive when the occurence of one prevents the occurence of the other.

eg. roll 2 dice.

event A : sum is less than 3
event B : sum is greater than 6

events A and B cannot happen together

- they are mutually exclusive.

A card is dealt

A : card is Heart.

B : card is Spade

- it clearly can't be both at the same time.

if two events are mutually exclusive the chance that at least one occurs is the ~~sh~~ sum of the chance of each.

Example.

what's the chance of the top card being either a H or a S. ?

chance $\frac{13}{52} = \frac{1}{4}$ that it is H.

$\frac{13}{52} = \frac{1}{4}$ S

it cannot be both $\Rightarrow$ mutally exclusive events

$P(H \text{ or } S) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$

Example

Roll 2 dice

what's the chance of at least one ⚀ ?

chance of ⚀ on red die is $\frac{1}{6}$

chance of ⚀ on white die is $1/6$

$$\frac{}{\text{total} \quad 1/3}$$

from the table of outcomes when we roll 2 dice

chance of at least one ⚀ is $\frac{11}{36}$.

the two events (⚀ on Red, ⚀ on white) are <u>not</u> mutually exclusive.

they <u>can</u> both occur at once (1,1)

addition rule <u>overcounts</u> ~~[illegible]~~ due to this.

– double counts the (1,1) outcome.

If two events are <u>not</u> mutually exclusive

the addition rule gives a result that is <u>too big</u>.

To correct for double counting, subtract the probability that both happen.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$P(\boxed{\cdot}$ on Red die <u>or</u> $\boxed{\cdot}$ on white die$)$

$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36}$$

$$= \frac{11}{36}$$

---

mutually exclusive vs independent

if A happens, B cannot.

knowing that A has happened does not change the probability that B will happen.

mutually exclusive events are <u>not</u> independent.

↳ knowing that one has happend tells you a <u>lot</u> (everything) about the probability of the other event.

$P(A)$.

$P(\sim A)$ "not A"

$P(A) + P(\sim A) = 1$. one of A or its complement must happen.

<u>Roll 4 dice.</u>

whats the chance that <u>at least one</u> ⚃ shows?

it's <u>not</u> $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

as the events ~~4 on a die~~

⚃ on 1st die, ⚃ on 2nd die, etc

are <u>not</u> mutually exclusive.

<u>can</u> list all possible outcomes

1	1	1	1
1	1	1	2
1	1	1	3
1	1	1	4
1	1	1	5
1	1	1	6
1	1	2	1
1	1	2	2
1	1	2	3

etc

but its tedious + error prone

Event A is "at least one ⚅ shows"

what is the complementary event?

- no ⚅'s show.

not a ⚅ on 1st die is independent of not a ⚅ on 2nd die.

$$P(\text{not a ⚅}) = \frac{5}{6}$$

$$P(\text{1st die is not ⚅ AND 2nd die is not ⚅}) = \frac{5}{6} \times \frac{5}{6}$$

as the events are independent

$$P(\text{no ⚅'s on 4 dice}) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$$

The event we're interested in is the complementary event to "no ⚅ on 4 dice"

=> prob. of at least one ⚅ on 4 dice

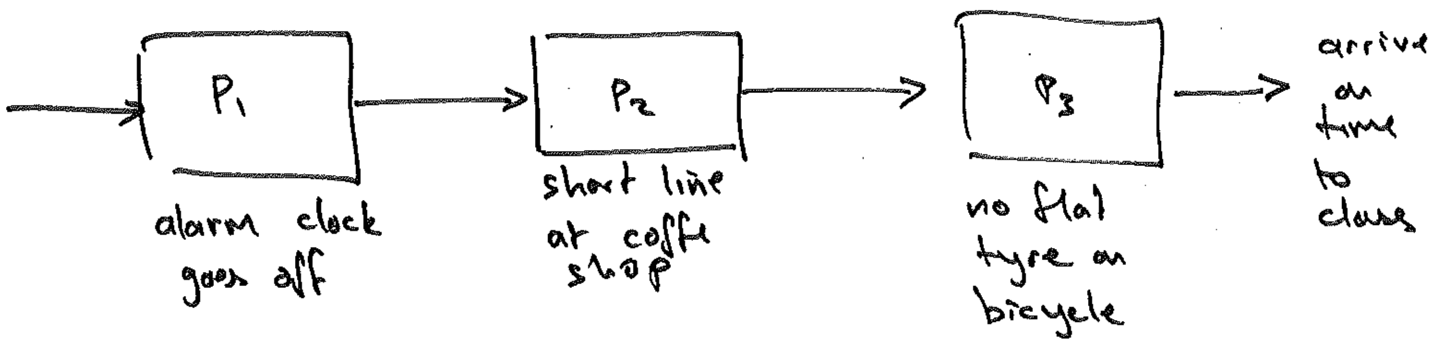
$$\text{is } 1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$$

$$= 1 - 0.482$$

$$= 0.518$$

if ever looking for chance of "at least one something"

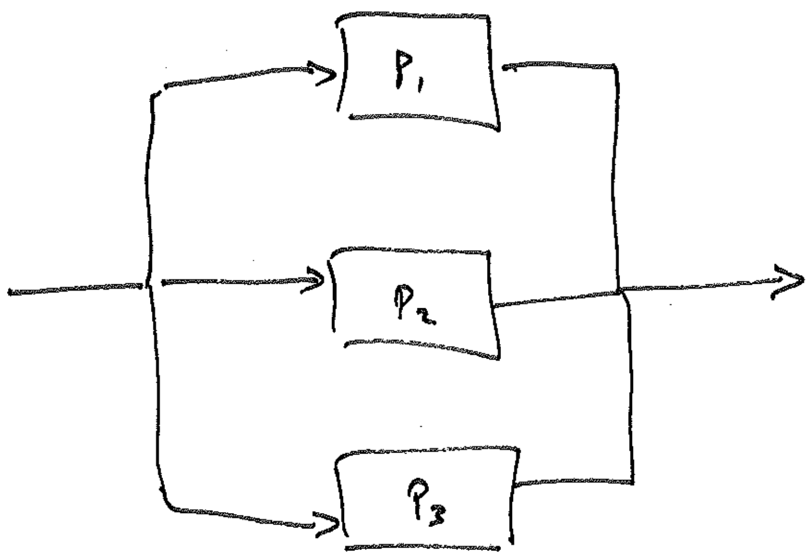
- compute chance of "no somethings"
- desired prob is 1 − chance of "no somethings"



if all components are independent

the system works when all the components work.

all the components work with prob. $P_1 \times P_2 \times P_3$



System works if <u>any one</u> of the components works.

Easiest to consider the case of the <u>system</u> not working

- when <u>all</u> the components do not work.
- the components are independent.

$\Rightarrow$ system doesn't work with prob

$$(1-P_1) \times (1-P_2) \times (1-P_3)$$

$\Rightarrow$ system <u>works</u> with prob $1-(1-P_1)(1-P_2)(1-P_3)$

Summary.

event A:

$$0 \leq P(A) \leq 1$$

opposite event

$$P(\text{not } A) = 1 - P(A)$$

addition

$$\underset{\text{(or both)}}{P(A \text{ or } B)} = P(A) + P(B) - \underbrace{P(A \text{ and } B)}_{=0}$$

if A, B are mutually exclusive

multiplication

$$P(A \text{ and } B) = P(A|B)\,P(B)$$

$$= P(B|A)\,P(A)$$

for independent events

$$= P(A)\,P(B)$$