# Probability - and how it relates to the Lottery.

Quantifying chance

- mathematics to calculate quantities associated with uncertain events

What do we mean by chance?

How do we interpret probabilities?

Die - if the die is balanced, when rolling it many times, each side should come up as often as any of the others.

So the side with 5 spots should come up $\frac{1}{6}$ of the time.

Definition: The chance of an event is the percentage of times the event is expected to happen when the process is repeated over and over independently and under the same conditions.

frequency definition

Aside: what about events that can't be repeated?

- what's the chance that it will rain tomorrow?
- will there be a colony on Mars by 2100?

→ subjective interpretation of probability

"probability as degree of belief"

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independently – outcome of one trial doesn't depend on earlier trials.

<u>Properties</u>.

if something is <u>impossible</u> it happens 0% of the time – its probability is <u>zero</u>.

if something <u>always</u> <u>happens</u>, it happens 100% of the time, its probability is <u>one</u>

⇒ chances are between 0% and 100%

0 and 1

chance of something happening <u>or</u> it not happening is 100%

eg. roll a 6 or not roll a 6.

one must happen.

chance of an event is 100% minus the chance of the opposite event.

<u>or</u>:

if an event has chance p of happening, the opposite has chance $1-p$

what's the chance of the coloured block I pull out of the bag being red?

what's the chance of it being not red (blue/green/orange)

list all the possible outcomes.

R
G
B
O

<u>count</u> the number that fit the condition.

P(not red). $\dfrac{3}{4}$. ← # fit. ← total # of outcomes.

Box contains red + blue balls.

one is drawn at random. ← each ball is equally likely to be chosen.

if red - win \$1.
blue - win \$0

Box 1 : 3 red , 2 blue

Box 2. : 30 red , 20 blue.

the probability of winning is the <u>same</u> in both cases.

- think back to the relative frequency definition.

- when drawing repeatedly, replacing the ball after each drawing ("same conditions"), where each ball has the same chance to be picked.

Box 1 : 3 out of 5 will be red : 60%.

Box 2 : 3 out of 5 will be red : 60%.

What's important is the <u>ratio</u>. $\dfrac{\#\text{ red}}{\text{total }\#}$ ← when outcomes are equally likely list all possible outcomes.

Box A.

R

R

R

B

B

**count** # that fit the condition

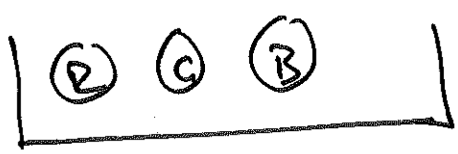
(ie 3).

**divide** by total # of outcomes.

$\frac{3}{5} = 60\%$

Drawing with and without replacement.

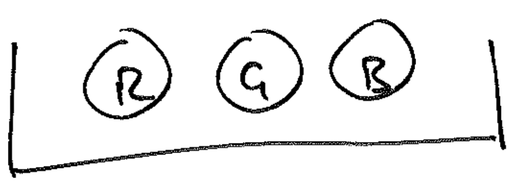
with replacement




after each draw we replace the ball

when make the next draw, the contents are the same

without replacement.




Imagine 1st draw is B


the chances of drawing each ball have changed.

need to introduce conditional probability

Shuffle a standard deck of cards.

What's the chance of Q♡ being in the second position.

- 52 possible positions
- interested in one of them.

$\Rightarrow$ prob is $\frac{1}{52}$.

Now: turn over 1st card, it's K♣.

What's the prob. that 2nd card is Q♡?

$\Rightarrow$ now, there are only 51 places the Q♡ could be, as we know it is not in the 1st position.

count: # possible positions = 51

# positions we're interested in = 1

prob = $\frac{1}{51}$

this is the <u>conditional probability</u> of the 2nd card being Q♡, <u>given</u> that 1st card was K♣.

↑

equivalent to "conditional on"

# Multiplication Rule.

R G B

Draw 2 balls
without replacement.

whats the probability of drawing

$1^{st}$ R

$2^{nd}$ G

from frequency definition:

imagine a large # of people each
drawing two balls.

About $1/3$ of them will draw R first

leaving G B

of these, $\frac{1}{2}$ of them will get G next.

if have 600 people to start

200 get R first

100 get R first then G second.

chances of drawing R, G are $\frac{100}{600} = \frac{1}{6}$

OR.

initially R G B.

each colour has equall chance of being drawn.

$$\text{prob } R = \frac{\#\text{ outcomes we're interested in}}{\#(\text{equally likely})\text{ outcomes.}}$$

$$= \frac{1}{3}$$

conditioned on drawing R, box contains

G B

prob of G $= \frac{1}{2}$

Combining these:

$\frac{1}{3}$ of the time we get the desired 1st outcome,

and of these, $\frac{1}{2}$ of the time we get the desired 2nd outcome.

ie $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of the time we get the desired outcome.

the probability that two events happen equals the probability that the first event happens times the probability that the second event will happen, given that the first has happened.

Example.

shuffle a deck.

what's prob. 1st card is 7 clubs
2nd card is QH ?

1st card : 7C can be in any of the 52 spots.
$\Rightarrow P(1^{st} \text{ card is } 7C) = \frac{1}{52}$.

2nd card : given that 1st card is 7C, the QH has 51 possible positions

$$P(2^{nd} \text{ card is } QH \mid 1^{st} \text{ card is } 7C) = \frac{1}{51}$$

$$P(1^{st} \text{ card is } 7C \text{ and } 2^{nd} \text{ card is } QH) = \frac{1}{52} \times \frac{1}{51}$$

I chose Y, R.

1st draw, what's the chance that <u>one</u> of my choices comes out?

$$\frac{2}{5}$$

2nd draw – assuming that one of mine came out first, what's the chances of the other coming out second?

$$\frac{1}{4}$$

Q: what's the chance of you winning the lottery?

chose 5 numbers from 1 - 69

and 1 number from 1 - 26.

$$\frac{5}{69} \times \frac{4}{68} \times \frac{3}{67} \times \frac{2}{66} \times \frac{1}{65} \times \frac{1}{26}$$

$= \cancel{0.000000006}$ a very small number....

ie your chances of winning do not really change if you buy a ticket.

$\cancel{4.07 \times 10^{-9}}$

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