Name:_____   Section: (day/time) _____

AMS5 - MIDTERM
Tuesday May 4th, 2010

A Normal Table is on the last page of this exam.

You must explain all answers and/or show working for full credit.

There are some **bonus questions.** You can get full marks on this exam without answering these questions. If you do give correct answers, you will get extra marks. You cannot score more than 100% on the exam, however.

You are reminded of the University's policy on Academic Misconduct.

1. *(2 marks)* There are 52 cards in a deck and 13 of them are hearts.

   (a) Four cards are dealt, one at a time, off the top of a well-shuffled deck. What is the chance that a heart turns up on the fourth card, but not before? Explain briefly.

   ② $$\underbrace{\frac{39}{52} \times \frac{38}{51} \times \frac{37}{50}}_{3\ non-hearts} \times \underbrace{\frac{13}{49}}_{heart\ on\ 4^{th}\ card}$$

   (b) A deck of cards is shuffled. You have to deal one card at a time until a heart turns up. You have dealt 3 cards and still have not seen a heart. What is the chance of getting a heart on the 4th card? Explain briefly.

   ① $\dfrac{13}{49}$ — 13 hearts

   — 49 cards left

2. *(6 marks)* Read the news report "Ambidextrous children 'more likely to be hyperactive' " printed at the end of this exam paper.

   (a) Does this describe a controlled experiment or an observational study? Why?

   ① observational study — children self-assign to ambidextrous/left/right handed.

1

(b) The 8,000 children were part of a "longitudinal study", that is, data is recorded about the same children over time. We are not told how the children were initially selected.

Would it affect the outcome of the study if the sample were biased, for example if it contained too many children from well-off families or too many girls? Explain your answer.

① No. The link is genetic and not environmental.

(c) What is the causal link between handedness and hyperactivity discussed in the article?

① weaker function in the brain's right hemisphere.

(d) Does the research prove this causal link? Explain briefly.

① No. The study shows association between mixed handedness + hyperactivity.

(e) The article says that

"Mixed handed children aged seven and eight were twice as likely as their right handed peers to have difficulties with language"

i. Which two conditional probabilities are being discussed here?

① P(language difficulty | mixed handed)

P(language difficulty | right handed).

ii. What is the relationship given in the article between these two conditional probabilities?

① P (language difficulty | mixed handed)

$= 2 \times$ P (language difficulty | right handed).

(f) [BONUS] *(2 marks)* If you are told that a randomly chosen seven year old has language difficulties, are they more or less likely to be right handed or ambidextrous? How much more/less likely? [Hint: either manipulate conditional probabilities (hard) or consider 1000 kids and count (easier)]

1000 kids, 1 in 100's ambidextrous. ⇒ 990 rt handed, 10 ambidextrous.

Assume p(language difficulty | amb.) = 1/10

then p (language difficulty | right) = 1/20

of the 990 rt handed kids $990 \times \frac{1}{20} = 49.5$ have language difficulties

10 amb. $10 \times \frac{1}{10} = 1$ has language difficulties.

⇒ if you know they have language difficulties   chance $\frac{49.5}{49.5 + 1} = 0.98$ of being right handed.

3. *(5 marks)* A small airline flies planes with seats for 4 passengers. Experience shows that for this airline, someone with a ticket will no-show 15% of the time.

(a) If the airline sells 4 tickets for a particular flight, what is the chance that the plane will not be full?

$$p(\text{not full}) = 1 - p(\text{full})$$
$$= 1 - p(4 \text{ out of } 4)$$
$$= 1 - \frac{4!}{4! \, 0!} (0.85)^4 (0.15)^0 = 0.478.$$

(b) If the airline sells 5 tickets, what is the chance that

i. someone will be "bumped"?

$$p(\text{bumped}) = p(5 \text{ out of } 5)$$
$$= \frac{5!}{5! \, 0!} (0.85)^5 = 0.44$$

ii. the flight will not be full?

$$p(\text{not full}) = 1 - p(\text{full})$$
$$= 1 - p(4 \text{ out of } 5) - p(5 \text{ out of } 5)$$
$$= 1 - \frac{5!}{4! \, 1!} (0.85)^4 \times 0.15 - \underbrace{0.44}_{\text{from above}}$$
$$= 0.1648.$$

A large airline flies planes with 200 seats, and they have a no-show rate of 2%.

(c) If they sell 200 tickets for a particular flight, what is the chance that the plane will be full?

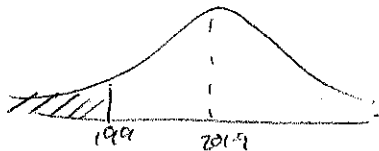$$- \text{if all 200 ticket holders show up} = (0.98)^{200}$$
$$= 0.0176.$$

(d) If they sell 206 tickets, what is the chance that there will be at least one empty seat?

$$EV = 206 \times 0.98 = 201.9$$
$$SE = \sqrt{206} \times (1-0) \times \sqrt{0.98 \times 0.02} \approx 2.$$

standard units $\frac{199 - 201.9}{2} = -1.45$

OR use binomial prob
$$1 - p(200 \text{ out of } 206)$$
$$- p(201 \text{ out of } 206)$$
$$\vdots$$
$$- p(206 \text{ out of } 206)$$

tail) area is $\frac{1}{2}(100 - 85.29)$ ≈ 7.4% .

$\underbrace{}_{\text{from table}}$

[BONUS] *(1 mark)* What is the chance that the Eyjafjallajökull volcano grounds all the planes and no one flies anywhere?

Erupts with chance 1 in 80 per year, grounds flights for 7 days.

So on a randomly chosen day, flights are grounded with prob $\frac{1}{80} \times \frac{7}{365} \approx 0.0002.$

3

4. *(8 marks)* The distribution by age in an unknown country is given in the table below. The class intervals include the left endpoint, but not the right one. The interval for 65 and over can be ended at 85.

Age	percentage	percent-per-year
0-5	7	1·4
5-15	14	1·4
15-20	7	1·4
20-30	14	1·4
30-35	7	1·4
35-45	15	1·5
45-55	14	1·4
55-65	10	1·0
65 and over	12	0·6

(a) Complete the table.

(b) Sketch the histogram on the graph on page 5. Label the axes.

(c) Are there more 21 year olds or more 61 year olds?

More 21 y/olds - area under 21-22 is large than under 61-62.
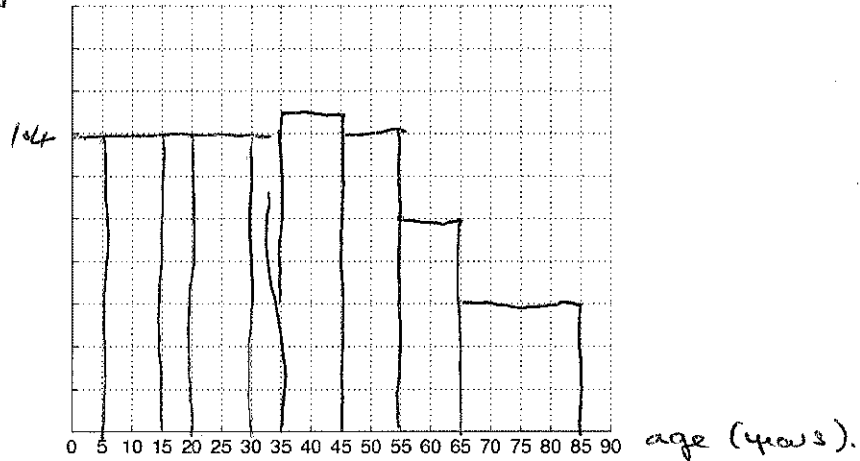
(d) What is the **median** age?

49% is up to age 35

median is $\frac{1}{15}$ of the way op the interval 35-45

is at age $35 + \frac{10}{15} = 35\frac{2}{3}$

4

[CONTINUED]

% per year

1.4 —

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 age (years).

(1) (e) Is the mean likely to be less than, greater than or close to the median? Explain your answer.

close to the median

Histogram is not very skewed.

(1) (f) Does the histogram appear to follow the normal curve? Explain briefly.

No - does not have Ω shape, especially at left.

(1) (g) The SD of age is approximately 24 years. What proportion of the data lies within 2 SD of the mean?

100% (interval is 24×4 = 96 years wide).

5. *(2 marks)* At the end of this exam is the first page of a letter from UCSC Chancellor George R. Blumenthal to UC President Mark Yudof. This letter was part of a discussion about reducing the UC's budget shortfall by reducing expenditure on staff and faculty.

Read the letter, paying particular attention to the section under the heading "Summary of Employee Comments".

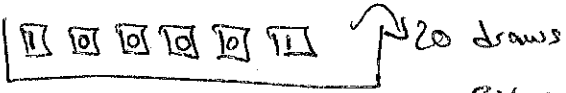(1) (a) How many comments were received overall?

650

(1) (b) How many respondents chose Option II? Explain your answer.

We do not know. The key phrase is "of those who responded with a specific option selection"

[TURN OVER]

6. *(3 marks)* A die will be rolled 20 times. The sum

number of ones rolled + number of sixes rolled

$\text{①} + \text{①}$ will be around _____ $6^{2}/_{3}$ _____, give or take _____ 2.1 _____ or
so. Show your working in the space below.

① $\boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{1}$ $\leftarrow$ 20 draws

$$EV = 20 \times \frac{1+0+0+0+0+1}{6} = 6^{2}/_{3}.$$

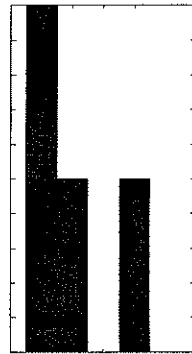$$SE = \sqrt{20} \times (1-0) \sqrt{\tfrac{2}{6} \times \tfrac{4}{6}} \doteq 2.1$$

7. *(3 marks)* Twenty draws are made at random with replacement from the box $\boxed{1}\ \boxed{1}\ \boxed{2}\ \boxed{4}$.
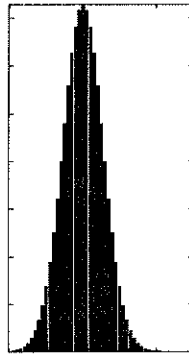One of the graphs below is the probability histogram for the average of the draws. Another
is the histogram for the numbers drawn, and the third is the histogram for the contents of
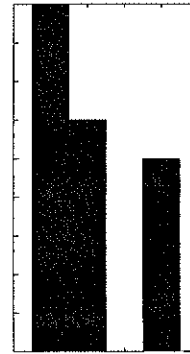the box. Which is which? Explain.

③

has same
shape as $\rightarrow$
box - twice
as likely to
be 1 than
2 or 4

(I)

hist. of
contents of
box

(II)

prob. hist for.
sum average of
draws

(III)

hist. of
numbers
drawn.

$\uparrow$
prob. hist for
sum resembles the
normal curve.

$\leftarrow$ there is fluctuation
away from the
chances $\left(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}\right)$.